Name of the Course : **CBCS B.Sc. Mathematical Sciences / B.Sc. (Prog.)**

Unique Paper Code : **42357602**

Name of the Paper : **DSE- Probability and Statistics**

Semester : **VI**

Duration : **3 Hours**

Maximum Marks : **75**

Attempt any four questions. All questions carry equal marks.

1. Let the random variable X be defined as the sum of faces in throwing two unbiased dice. Write the probability distribution of X. Find the cumulative distribution function of X. Also find mean deviation about mean and variance. Further find the probability that the sum is
(i) greater than 9 (ii) neither 5 nor 7.

   Let X have the probability density function

$$f(x) = \begin{cases} \frac{3x^2}{8} & 0 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

   Find the distribution function of X and the probability density function of $Y = X^3$.

2. Let

$$f(x) = \begin{cases} ke^{-3x} & \text{for } 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

   be the probability density function of a random variable X. Evaluate k. Find 25^{th} percentile, first four moments, mode and the moment generating function of X.

3. Suppose that the CGPA scores of a large population of first-year college students are approximately normally distributed with mean 2.5 and standard deviation 0.7. What fraction of the students will have a CGPA more than 3.0?
   If students possessing a CGPA score less than 1.8 are not promoted to next year, what percentage of the students will not be promoted to next year?
   Suppose that three students are randomly selected from the first-year student body. What is the probability that all three will possess a CGPA score more than 3.0?

   Let X be a geometric random variable with parameter $p = 0.4$ and let $Y = 2X$-1. Find $E(Y), Var(Y)$ and the moment generating function of Y.

4. Let X and Y be the random variables with the joint probability density function

$$f(x,y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

i) Evaluate $P(X < 1, Y > 5)$
ii) Evaluate $P(X + Y < 3)$
iii) Find the marginal densities of X and Y.
iv) For any $y > 0$, find the conditional density function of X given that $Y = y$.
v) Evaluate $E(X - Y)$ and $Var(X - Y)$.

5. Let the random variables X and Y have the joint probability density function

$$f(x,y) = \begin{cases} 3x & 0 < y < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? If not, then find $\mu_{Y|x}$ and $\mu_{X|y}$. Also, compute the correlation coefficient between X and Y.

6. Let X be a random variable such that $P(X \leq 0) = 0$ and let $\mu = E(X)$ exist. Show that $P(X \geq 2\mu) \leq \frac{1}{2}$.

Calculate the rank correlation coefficient r_S for the following data representing the statistics grades, x, and psychology grades, y, of 18 students:

x: 78, 86, 49, 94, 53, 89, 94, 71, 70, 97, 74, 53, 58, 62, 74, 74, 70, 74
y: 80, 74, 63, 85, 55, 86, 90, 84, 71, 90, 85, 71, 67, 64, 69, 71, 67, 71